The center of the Taylor circle

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Abstract

This paper answers a question from Paul Yiu to group *Hyacinthos*. How give a synthetic proof that the center of the Taylor circle of a given triangle is the Spieker center of its orthic triangle ? More properties of the orthic triangle, its medial triangle and the Taylor circle can be found, with synthetic proofs, in [1] pp. 26–41.

1 Notations

We denote

- ABC any triangle in the plane;
- H the orthocentrer of ABC;
- $H_AH_bH_c$ the orthic triangle of ABC, i.e. both cevian and pedal triangle of H;
- $A'B'C'$ the medial triangle of $H_AH_bH_c$, i.e. A' is the midpoint between H_b and H_c, B' and C' cyclically;
- H_{ab} the orthogonal projection of H_a on the line (AB), H_{ac}, H_{ba}, H_{bc}, H_{ca}, H_{cb} cyclically.

The following results are not proved.

The lines (H_bH_c) and (BC) are antiparallel with respect to (AB) and (AC).

The perpendicular bisector of $[H_bH_c]$ passes through the midpoint between B and C.

Both results become of the concyclicity of B, C, H_b, H_c on the circle with diameter $[BC]$ and center the midpoint between B and C.

2 Parallel lines

Proposition 1
The line $(H_{ba}H_{ca})$ and (BC) are parallel (cf. fig. 1).

Proof : H_{ba} and H_{ca} are, in AH_bH_c, the feet of the altitudes, so the lines $(H_{ba}H_{ca})$ and (H_bH_c) are antiparallel with respect to (AB) and (AC).
The result becomes because (H_bH_c) and (BC) are also antiparallel with respect to (AB) and (AC).

□

Proposition 2
The perpendicular bisector of $[H_{ba}H_{ca}]$ is the A'-angle bisector of $A'B'C'$ (cf. fig. 1).

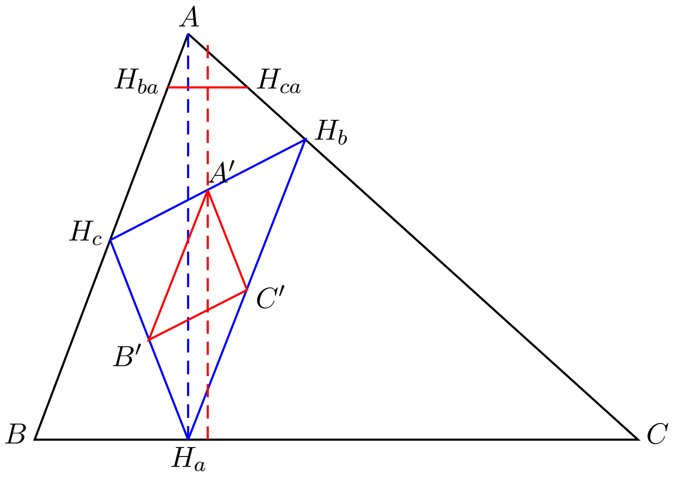

Figure 1:

Proof : The perpendicular bisector of $[H_{ba}H_{ca}]$ passes through A' and is perpendicular to $(H_{ba}H_{ca})$ so to (BC) : it is the parallel, through A' to the altitude (AH_a). This line is the H_a-angle bisector in the orthic triangle $H_aH_bH_c$. The resultes becomes obviously.

□

We have :

- The perpendicular bisectors of $[H_{ba}H_{ca}]$, $[H_{cb}H_{ab}]$, $[H_{ac}H_{bc}]$ are the angle bisectors of $A'B'C'$, they are concurent at its incentrer [1], i.e. the Spieker point of the orthic triangle.
- The points H_{ba}, H_{ca}, H_{cb}, H_{ab}, H_{ac}, H_{bc} are concyclic on the Taylor circle of ABC, so the perpendicular bisectors of $[H_{ba}H_{ca}]$, $[H_{cb}H_{ab}]$, $[H_{ac}H_{bc}]$ are concurent at the center of this circle.

The center of the Taylor circle is the Spieker center of the orthic triangle.

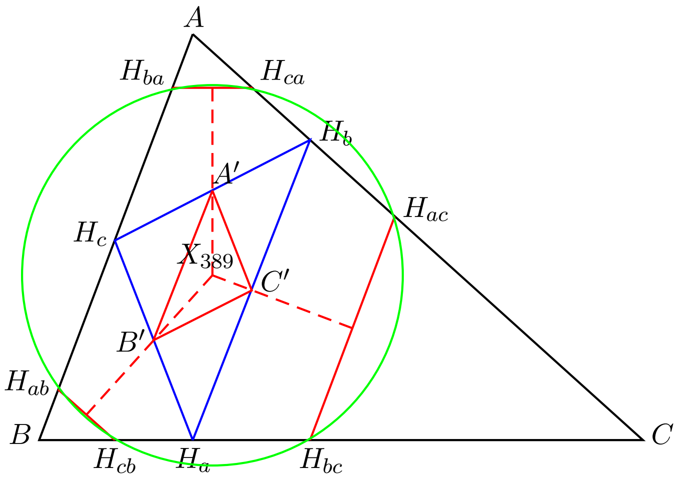

Figure 2: The Taylor circle

References

[1] Yvonne et René SORTAIS. *La géométrie du triangle.* HERMANN, Paris, 1987.

[1] with ABC acute angled, else they are concurent at one of its excenters.